

Contents

1	The Wireless Communication Environment	1
2	Modelling of Wireless Systems	2
3	System Model for Narrowband Signals	5
4	Rayleigh Fading Wireless Channel	6
4.1	Baseband Model of a Wireless System	7
5	Basics of Wireless Channel Modelling	10
5.1	Maximum Delay Spread $\sigma_{\tau, \max}$	11
5.2	RMS Delay Spread $\sigma_{\text{RMS}}^{\tau}$	12
5.3	RMS Delay Based on Average Power Profile	14
6	Average Delay Spread in Outdoor Cellular Channels	15
7	Coherence Bandwidth in Wireless Communications	16
8	Relation Between ISI and Coherence Bandwidth	21
9	Doppler Fading in Wireless Systems	24
9.1	Doppler Shift Computation	24
10	Doppler Impact on a Wireless Channel	26
11	Coherence Time of the Wireless Channel	29

1 The Wireless Communication Environment

In conventional wireline communication systems, there is a single signal-propagation path between the transmitter and the receiver, which is constrained by the propagation medium such as a coaxial cable or a twisted pair.

However, in wireless systems, the signal can reach the receiver via direct, reflected, and scattered paths as shown in Figure 1. As a result, at the receiver, there is a superposition of multiple copies of the transmitted signal. These signal copies experience different attenuations, delays, and phase shifts arising from the varied propagation distances and properties of the scattering media.

Hence, at the wireless receiver, there is interference of signals received from these multiple propagation paths, which is termed **multipath interference**.

The multipath interference, in turn, results in an amplification or attenuation of the net received signal power observed at the receiver, and this variation in the received signal strength arising from the multipath propagation phenomenon is termed **multipath fading** or simply **fading**.

Strong destructive interference at the receiver is referred to as a **deep fade**, and such a condition may result in temporary failure of communication due to a severe drop in the SNR at the receiver.

This phenomenon of fading is the critical difference between the wireline and wireless communication systems, which causes a radical paradigm shift in the nature of wireless communications, necessitating the development of a fundamentally novel approach to design wireless systems.

One of the main objectives, therefore, in wireless-system design is to develop schemes to combat fading and ensure reliability of signal reception in wireless communication systems.

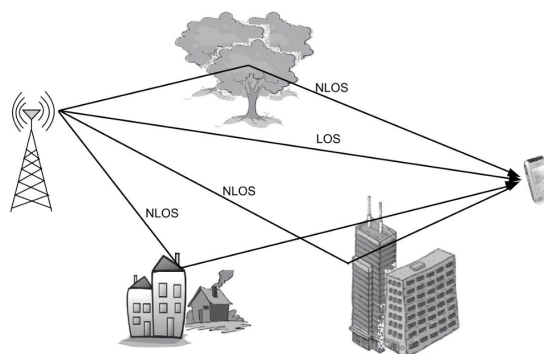


Figure 1: Schematic of the wireless-propagation environment

2 Modelling of Wireless Systems

To gain a better understanding of the nature of the wireless environment and quantitatively analyze the performance of wireless communication systems, we need to develop accurate analytical models to characterize them. These models provide the theoretical basis for evaluating various wireless communication schemes and systems.

Consider the transmitted passband wireless signal $s(t)$ given by

$$s(t) = \Re \{ s_b(t) e^{j2\pi f_c t} \} \quad (1)$$

where $s_b(t)$ is the complex baseband representation of the transmitted signal, and f_c is the carrier frequency used for transmission.

For simplicity, assume that the wireless channel is time-invariant and consists of L multipath components. Each i^{th} path has two main characteristics:

- Attenuation factor: a_i
- Path delay: τ_i

From linear system theory, the impulse response of a path that attenuates the signal by a_i and delays it by τ_i is

$$h_i(\tau) = a_i \delta(\tau - \tau_i)$$

Thus, the total channel impulse response (CIR) for L multipath components is

$$h(\tau) = \sum_{i=0}^{L-1} a_i \delta(\tau - \tau_i) \quad (2)$$

This representation is called the **tapped delay line model**, as the received signal is a sum of several delayed versions of the transmitted signal.

From the theory of LTI systems, the received passband signal is obtained by convolving the transmitted signal $s(t)$ with the CIR $h(t)$:

$$y(t) = s(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) s(t - \tau) d\tau$$

Substituting Eq. (2) and applying the sifting property of the delta function:

$$y(t) = \sum_{i=0}^{L-1} a_i s(t - \tau_i)$$

Using Eq. (1) for $s(t)$:

$$y(t) = \Re \left\{ \sum_{i=0}^{L-1} a_i s_b(t - \tau_i) e^{j2\pi f_c(t - \tau_i)} \right\}$$

Factoring out $e^{j2\pi f_c t}$:

$$y(t) = \Re \left\{ \left[\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t - \tau_i) \right] e^{j2\pi f_c t} \right\}$$

The equivalent complex baseband received signal is therefore

$$y_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t - \tau_i) \quad (3)$$

From Eq. (3), each received baseband signal copy is characterized by:

1. Attenuation factor a_i
2. Path delay τ_i
3. Phase shift $e^{-j2\pi f_c \tau_i}$

The time difference between the earliest and latest arriving copies is called the **delay spread** T_m . For a **narrowband** channel, $T_m \ll T$ (symbol time), which simplifies analysis.

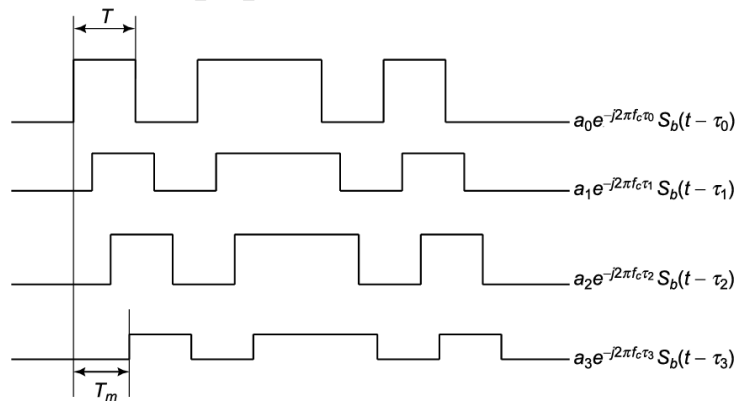


Figure 2: Multipath signal components at the receiver

Example 1

Consider a wireless signal with a carrier frequency of $f_c = 850$ MHz, which is transmitted over a wireless channel that results in $L = 4$ multipath components at delays of 201, 513, 819, 1223 ns and corresponding to received signal amplitudes of 1, 0.6, 0.3, 0.2, respectively. Derive the expression for the received baseband signal $y_b(t)$ if the transmitted baseband signal is $s_b(t)$.

Solution: From Eq. (3), the received baseband signal is given by

$$y_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t - \tau_i) \quad (4)$$

We compute $a_i e^{-j2\pi f_c \tau_i}$ for $i = 0, 1, 2, 3$:

$$a_0 e^{-j2\pi f_c \tau_0} = 1 \cdot e^{-j2\pi(850 \times 10^6)(201 \times 10^{-9})}$$

$$\approx 0.59 + j 0.81,$$

$$a_1 e^{-j2\pi f_c \tau_1} = 0.6 \cdot e^{-j2\pi(850 \times 10^6)(513 \times 10^{-9})}$$

$$\approx 0.57 - j 0.19,$$

$$a_2 e^{-j2\pi f_c \tau_2} = 0.3 \cdot e^{-j2\pi(850 \times 10^6)(819 \times 10^{-9})}$$

$$\approx 0.18 - j 0.24,$$

$$a_3 e^{-j2\pi f_c \tau_3} = 0.2 \cdot e^{-j2\pi(850 \times 10^6)(1223 \times 10^{-9})}$$

$$\approx -0.19 + j 0.06.$$

Therefore,

$$\begin{aligned} y_b(t) = & (0.59 + j 0.81) s_b(t - 201 \times 10^{-9}) \\ & + (0.57 - j 0.19) s_b(t - 513 \times 10^{-9}) \\ & + (0.18 - j 0.24) s_b(t - 819 \times 10^{-9}) \\ & + (-0.19 + j 0.06) s_b(t - 1223 \times 10^{-9}). \end{aligned} \quad (5)$$

Under the *narrowband assumption*, the received signal is

$$y(t) = \sum_{i=0}^3 a_i e^{-j2\pi f_c \tau_i} s_b(t) \approx (1.14 + j 0.44) s_b(t) \quad (6)$$

Thus, the net received baseband signal can be expressed as

$$y_b(t) = h s_b(t), \quad \text{where } h = 1.14 + j 0.44 \quad (7)$$

Here, h is the *channel coefficient*.

3 System Model for Narrowband Signals

A fairly simplistic approximation of the above baseband system model can be arrived at by employing the narrowband signal approximation, which can be stated as follows. For a sufficiently narrowband signal $s_b(t)$, the different delayed components $s_b(t - \tau_i)$ are approximately equal to each other, i.e.,

$$s_b(t - \tau_i) \approx s_b(t)$$

Such a wireless channel is also known as a *flat-fading wireless channel*. Substituting the above approximation into the general baseband model, we obtain

$$y_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t) = h s_b(t) \quad (4)$$

where

$$h = a e^{j\phi} = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} \quad (5)$$

Here, h is termed the *complex fading channel coefficient*. It represents a complex attenuation factor $a e^{j\phi}$.

Key Points

- The received signal power depends on $|a e^{j\phi}|$.
- **Example:** For a two-component multipath channel with identical magnitude but exactly out-of-phase:

$$a_0 = a_1, \quad e^{-j2\pi f_c \tau_0} = -e^{-j2\pi f_c \tau_1}$$

we have:

$$y_b(t) = 0$$

This results in 0 received signal power ($-\infty$ dB), and the channel is in a *deep fade*.

- The parameter h is also called the *complex baseband fading coefficient* or simply the *fading coefficient*.
- The narrowband approximation does **not** hold for wideband systems such as CDMA.
- Narrowband assumption essentially implies that:
 - The carrier phase is sensitive to delay spread.
 - The baseband signal is not sensitive to delay spread.
- The approximation is usually valid when:

$$\text{Bandwidth of the transmitted signal} \ll f_c$$

4 Rayleigh Fading Wireless Channel

The complex fading coefficient h can be expressed in terms of its real and imaginary components as:

$$h = ae^{j\phi} = \sum_{i=0}^{L-1} (x_i + jy_i) = X + jY$$

Here, X and Y are the real and imaginary components of the fading coefficient $ae^{j\phi}$, obtained from the summation of a large number of random multipath components x_i, y_i . In a rich urban setting with many scatterers, it is reasonable to model X and Y as **random variables**. A simple model assumes:

- X and Y are Gaussian distributed.
- X and Y are uncorrelated and hence independent.

By the **Central Limit Theorem**, as $L \rightarrow \infty$, the sum of many independent components approaches a Gaussian distribution. Assuming zero mean and variance $\frac{1}{2}$:

$$X, Y \sim \mathcal{N}\left(0, \frac{1}{2}\right)$$

The joint PDF of X and Y is:

$$f_{X,Y}(x, y) = \frac{1}{\pi} e^{-(x^2+y^2)} \quad (1)$$

Transformation to Amplitude and Phase

Let:

$$x = a \cos \phi, \quad y = a \sin \phi$$

Using the multivariate PDF transformation, we get:

$$f_{A,\Phi}(a, \phi) = \frac{1}{\pi} e^{-a^2} J_{X,Y} \quad (2)$$

where $J_{X,Y}$ is the Jacobian determinant:

$$J_{X,Y} = \begin{vmatrix} \cos \phi & \sin \phi \\ -a \sin \phi & a \cos \phi \end{vmatrix} = a$$

Thus:

$$f_{A,\Phi}(a, \phi) = \frac{a}{\pi} e^{-a^2} \quad (3)$$

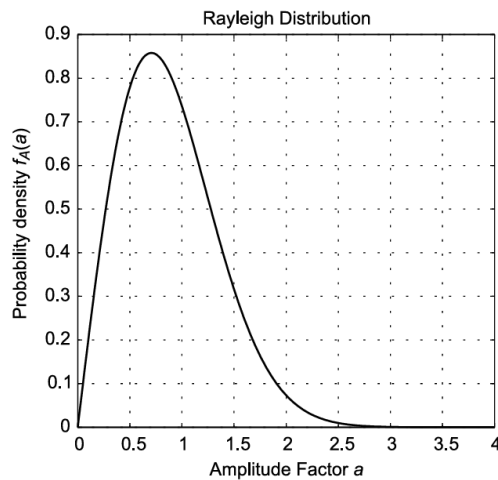


Figure 3: Rayleigh density for amplitude factor a .

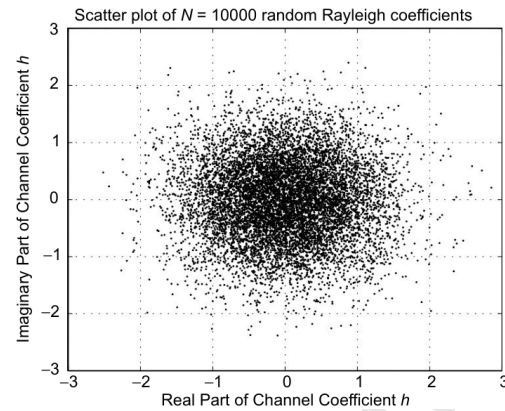


Figure 4: Scatter plot of real and imaginary components of Rayleigh fading coefficient.

Marginal Distributions

Amplitude distribution:

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\Phi}(a, \phi) d\phi = 2ae^{-a^2}, \quad a \geq 0 \quad (4)$$

Phase distribution:

$$f_{\Phi}(\phi) = \int_0^{\infty} f_{A,\Phi}(a, \phi) da = \frac{1}{2\pi}, \quad -\pi < \phi \leq \pi \quad (5)$$

The amplitude distribution $f_A(a)$ is the well-known **Rayleigh distribution**. The average power of the Rayleigh fading channel coefficient h is:

$$E\{|h|^2\} = E\{a^2\} = E\{X^2 + Y^2\} = 1 \quad (6)$$

Key Observations

- Amplitude a is Rayleigh distributed.
- Phase ϕ is uniformly distributed in $(-\pi, \pi]$.
- a and ϕ are independent random variables.
- For a given a , all phase values are equally likely.

4.1 Baseband Model of a Wireless System

- **Transmitted and received symbols** — $x(k)$ is the k -th transmitted symbol, and $y(k)$ is the k -th received symbol.

- **Channel coefficient** h — Represents the effect of multipath fading in a Rayleigh fading channel.
- **Baseband model** —

$$y(k) = h x(k) + n(k)$$

where $n(k)$ is additive white Gaussian noise (AWGN).

- **Noise assumption** — Noise is usually assumed to have unit power:

$$E\{|n(k)|^2\} = 1$$

Scaling the system by a constant does not change the SNR.

- **Example – BPSK modulation** — Symbols are $\pm\sqrt{P}$, where P is the average symbol power.
- **Non-fading (AWGN) channel** — Without fading, $h = 1$:

$$y(k) = x(k) + n(k)$$

- **Naming convention** — Wireless with fading $\rightarrow$ Rayleigh fading channel. Wireline without fading $\rightarrow$ AWGN channel.

Example 2. PDF of the Channel Power Gain in a Rayleigh Fading Channel

Problem: Derive the probability density function of the channel power gain

$$g = \frac{a^2}{E\{a^2\}},$$

where a is the magnitude of the Rayleigh fading channel coefficient, and $E\{a^2\} = 1$ (so $g = a^2$).

Solution: From the Rayleigh model, the amplitude a has pdf

$$f_A(a) = 2a e^{-a^2}, \quad a \geq 0,$$

and $E\{a^2\} = 1$ (normalized average power).

Define the transformation

$$g = w(a) = a^2, \quad a \in [0, \infty), \quad \Rightarrow \quad a = w^{-1}(g) = \sqrt{g}, \quad \left| \frac{da}{dg} \right| = \frac{1}{2\sqrt{g}}.$$

By the pdf transformation rule,

$$f_G(g) = f_A(\sqrt{g}) \left| \frac{da}{dg} \right| = (2\sqrt{g} e^{-g}) \left(\frac{1}{2\sqrt{g}} \right) = e^{-g}, \quad g \geq 0.$$

Checks:

$$\int_0^{\infty} f_G(g) dg = \int_0^{\infty} e^{-g} dg = 1 \quad (\text{valid pdf}),$$

$$\mathbb{E}\{g\} = \int_0^{\infty} g e^{-g} dg = 1 \quad (\text{unit mean}).$$

Result: The channel power gain is exponentially distributed with unit mean:

$$f_G(g) = e^{-g}, \quad g \geq 0, \quad F_G(g) = 1 - e^{-g}, \quad g \geq 0.$$

Example 3 (Probability of Received Power Above a Threshold). In the Rayleigh fading channel, let the transmit power be

$$P_t(\text{dB}) = 20 \text{ dB}.$$

find the probability that the received power is greater than

$$P_r(\text{dB}) = 10 \text{ dB}.$$

Step 1: Convert to linear scale. From the relation $P(\text{dB}) = 10 \log_{10} P$, the linear transmit and receive powers are:

$$P_t = 10^{20/10} = 10^2 = 100,$$

$$P_r = 10^{10/10} = 10^1 = 10.$$

Step 2: Relate P_r to the channel power gain g . For a given power gain g , the received power is

$$P_r = gP_t.$$

The condition $P_r > 10$ implies:

$$gP_t > 10 \quad \Rightarrow \quad g > \frac{10}{P_t} = \frac{10}{100} = \frac{1}{2}.$$

Step 3: Compute the probability. For a normalized Rayleigh channel, the channel power gain g has pdf:

$$f_G(g) = e^{-g}, \quad g \geq 0.$$

Thus,

$$\mathbb{P}(g > \frac{1}{2}) = \int_{1/2}^{\infty} e^{-g} dg = [-e^{-g}]_{g=1/2}^{\infty} = e^{-1/2} \approx 0.61.$$

Final Answer: The probability that the received power exceeds 10 dB is about 61%.

5 Basics of Wireless Channel Modelling

As we had seen in the previous chapter, the fading wireless channel comprises several multipath components arising from the presence of multiple Non-Line-Of-Sight (NLOS) signal-propagation paths. These NLOS components arise from the scattering effects of objects in the wireless environment such as buildings, trees, vehicles, water bodies, etc.

In this section, we aim to develop a systemic framework to study the nature of the wireless channel, especially with respect to key properties such as inter-symbol interference and time-varying characteristics.

The impulse response of the standard multipath wireless channel can be modelled as

$$h(t) = \sum_{i=0}^{L-1} a_i \delta(t - \tau_i) \quad (8)$$

where each $\delta(t - \tau_i)$ corresponds to delaying the signal by τ_i , and a_i is the attenuation associated with the i th path. The quantity L denotes the number of multipath components.

From the above impulse response, one can define the multipath power profile as

$$\phi(t) = \sum_{i=0}^{L-1} |a_i|^2 \delta(t - \tau_i) = \sum_{i=0}^{L-1} g_i \delta(t - \tau_i) \quad (9)$$

where $g_i = |a_i|^2$ is the power gain of the i th path.

Table 1: Path gains and associated delays for an $L = 4$ multipath channel

Path Index i	Gain $g_i = a_i ^2$	Delay τ_i
0	$ a_0 ^2$	τ_0
1	$ a_1 ^2$	τ_1
2	$ a_2 ^2$	τ_2
3	$ a_3 ^2$	τ_3

Thus, the total power received in a multipath wireless channel occurs over a spread of time referred to as the *delay spread*. This spread of the arriving power at the wireless receiver is in contrast to that of a wireline channel, in which all the power is received at a single time instant due to the presence of only a single propagation path. The delay spread of a wireless channel is a key parameter that characterizes the nature of the wireless environment and is denoted by σ_τ .

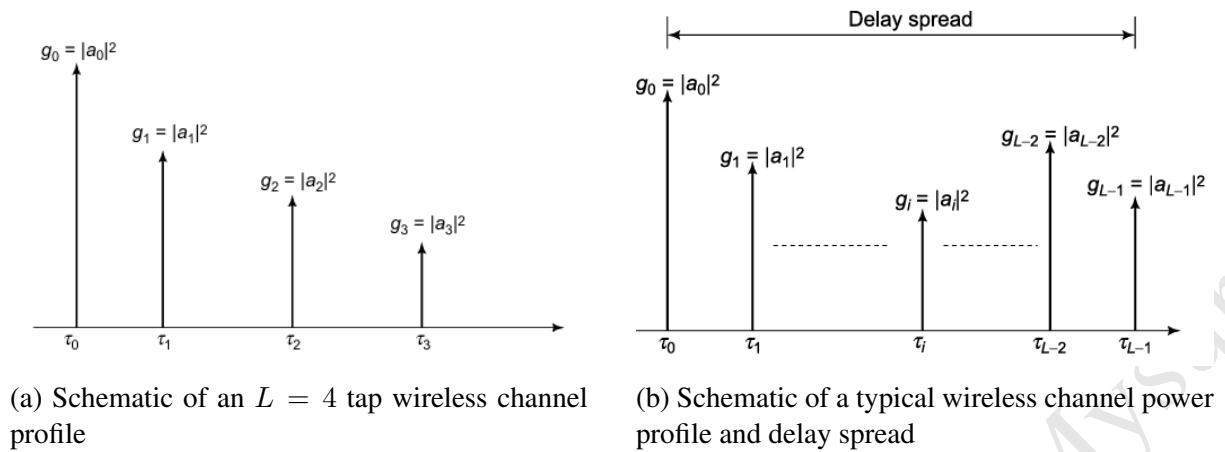


Figure 5: Wireless channel characteristics

5.1 Maximum Delay Spread $\sigma_{\tau, \max}$

Definition. The maximum delay spread is the time gap between the first and last arriving multipath components:

$$\sigma_{\tau, \max} = \tau_{L-1} - \tau_0, \quad (10)$$

where τ_0 is the first-path delay, τ_{L-1} is the last-path delay, and L is the number of multipath components.

Meaning

- Measures how “spread out” in time the received energy is due to multipath.
- Larger $\sigma_{\tau, \max} \Rightarrow$ richer scattering, larger differential path delays, and higher ISI risk.

Key property.

- Depends only on the *difference* $\tau_{L-1} - \tau_0$, not on absolute delays.
- Distance to the base station increases absolute propagation delay for all paths nearly equally, but does not change $\sigma_{\tau, \max}$.

Examples.

- *Single path:* If only one path with delay τ_0 exists,

$$\sigma_{\tau, \max} = \tau_0 - \tau_0 = 0.$$

- *Multiple paths:* If $\tau_0 = 2 \mu s$ and $\tau_{L-1} = 7 \mu s$,

$$\sigma_{\tau, \max} = 7 - 2 = 5 \mu s.$$

Interpretation.

- Higher $\sigma_{\tau, \max}$ indicates a more complex multipath environment and potential inter-symbol interference (ISI).

Example 1: Maximum Delay Spread

Question: Consider an $L = 4$ multipath channel with the delays and last arriving paths, given as $\tau_0 = 0 \mu s$ and $\tau_{L-1} = 5 \mu s$ corresponding to the first and last arriving paths, respectively. Such a wireless-channel power profile is shown schematically in Figure 6. What is the maximum delay spread corresponding to this wireless channel?

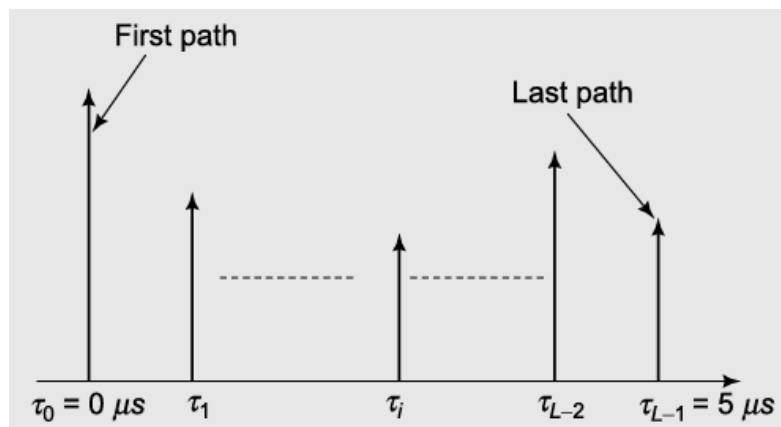


Figure 6: Power profile for Example 1

Solution: The maximum delay spread is given by: $\sigma_{\tau, \max} = \tau_{L-1} - \tau_0$

Substituting the given values: $\sigma_{\tau, \max} = 5 \mu s - 0 \mu s = 5 \mu s$

Thus, the maximum delay spread for this channel is: $5 \mu s$

5.2 RMS Delay Spread $\sigma_{\text{RMS}}^{\tau}$

In typical wireless channels, later-arriving paths often have much lower power due to longer travel and weaker reflections (see Fig. 7). Consequently, the maximum delay spread $\sigma_{\tau, \max}$ can be large even when the late paths carry negligible energy. Since $\sigma_{\tau, \max}$ does not weight delays by their power, it may not reflect the true spread of significant energy. The RMS delay spread remedies this by weighting each path's delay in proportion to its power.

Discrete multipath power profile. Consider the L -tap power profile from (9) with

$$g_i = |a_i|^2, \quad 0 \leq i \leq L - 1,$$

where g_i is the power gain of the i th path. Define the power fraction

$$b_i \triangleq \frac{g_i}{g_0 + g_1 + \cdots + g_{L-1}}, \quad b_i > 0, \quad \sum_{i=0}^{L-1} b_i = 1. \quad (11)$$

The (power-weighted) mean delay is

$$\tau = \sum_{i=0}^{L-1} b_i \tau_i \quad (12)$$

$$= \sum_{i=0}^{L-1} \frac{g_i}{\sum_{j=0}^{L-1} g_j} \tau_i \quad (13)$$

$$= \frac{\sum_{i=0}^{L-1} g_i \tau_i}{\sum_{j=0}^{L-1} g_j}. \quad (14)$$

The RMS delay spread is the (power-weighted) standard deviation of delays:

$$(\sigma_{\text{RMS}}^\tau)^2 = \sum_{i=0}^{L-1} b_i (\tau_i - \tau)^2, \quad (15)$$

$$\sigma_{\text{RMS}}^\tau = \sqrt{\sum_{i=0}^{L-1} b_i (\tau_i - \tau)^2} \quad (16)$$

$$= \sqrt{\frac{\sum_{i=0}^{L-1} g_i (\tau_i - \tau)^2}{\sum_{i=0}^{L-1} g_i}} \quad (17)$$

$$= \sqrt{\frac{\sum_{i=0}^{L-1} |a_i|^2 (\tau_i - \tau)^2}{\sum_{i=0}^{L-1} |a_i|^2}}. \quad (18)$$

Because each delay is weighted by its path power, weak trailing paths contribute very little, making σ_{RMS}^τ robust to spurious low-power components.

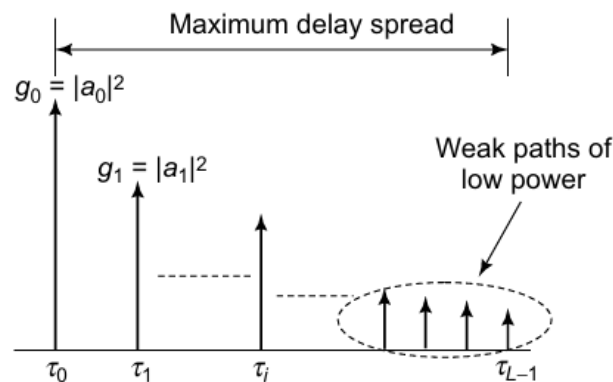


Figure 7: Power profile with weak trailing paths of very low power.

5.3 RMS Delay Based on Average Power Profile

In this section, we generalize the delay spread metric to a **continuous average power profile**.

Step 1: Average power at a given delay Let $|h(\tau)|^2$ be the instantaneous power of the channel at delay τ . The *average* power at delay τ is:

$$\phi(\tau) = \mathbb{E}[|h(\tau)|^2]$$

Here: - $\phi(\tau)$ tells us, on average, how much power arrives with delay τ . - The expectation $\mathbb{E}[\cdot]$ can be over **time** (for a single user) or over an **ensemble** of users in a given area.

Step 2: Fractional power distribution We normalize $\phi(\tau)$ so that it becomes like a probability density function:

$$f(\tau) = \frac{\phi(\tau)}{\int_0^\infty \phi(\tau) d\tau}$$

Here: - $f(\tau)$ is the **fractional power** at delay τ . - This means $f(\tau) \Delta\tau$ is the fraction of total power arriving between delays τ and $\tau + \Delta\tau$.

Step 3: Mean delay The *average delay* $\bar{\tau}$ is found by weighting each delay by its fractional power:

$$\bar{\tau} = \int_0^\infty \tau f(\tau) d\tau = \frac{\int_0^\infty \tau \phi(\tau) d\tau}{\int_0^\infty \phi(\tau) d\tau}$$

This gives us the "center of mass" of the power-delay profile.

Step 4: RMS delay spread The RMS delay spread measures how far the delays are spread out around the mean delay:

$$\sigma_{\text{RMS},\tau} = \sqrt{\int_0^\infty (\tau - \bar{\tau})^2 f(\tau) d\tau}$$

Substituting $f(\tau)$ in terms of $\phi(\tau)$:

$$\sigma_{\text{RMS},\tau} = \sqrt{\frac{\int_0^\infty (\tau - \bar{\tau})^2 \phi(\tau) d\tau}{\int_0^\infty \phi(\tau) d\tau}}$$

This formula: - Weighs delays according to their power. - Ignores insignificant weak paths automatically. - Gives a realistic measure of the spread of useful signal energy over time.

6 Average Delay Spread in Outdoor Cellular Channels

In outdoor cellular systems, the typical cell radius is in the range of **1–5 km**, meaning signal propagation distances are usually on the order of a few kilometres.

Example scenario: Consider two propagation paths as shown in Figure: - **Direct path length:** $d_0 = 2$ km - **Scattered path length:** $d_1 = 3$ km

The corresponding propagation delays are:

$$\tau_0 = \frac{d_0}{c}, \quad \tau_1 = \frac{d_1}{c}$$

where $c = 3 \times 10^8$ m/s is the speed of light.

Maximum delay spread:

$$\sigma_{\max, \tau} = \Delta\tau = \tau_1 - \tau_0 = \frac{\Delta d}{c}$$

Substituting the values:

$$\sigma_{\max, \tau} = \frac{3000 \text{ m} - 2000 \text{ m}}{3 \times 10^8 \text{ m/s}} = 3.33 \mu\text{s}$$

- This is not an *exact* calculation, but an **order-of-magnitude** estimate.
- In typical outdoor cellular scenarios (distances of a few km), **delay spreads are usually in the range 1–3 μs .**
- For indoor environments with distances of about 10 m, **typical delay spreads are much smaller: 10–50 ns.**
- Such estimates are valuable for the practical design of **3G/4G wireless communication systems.**

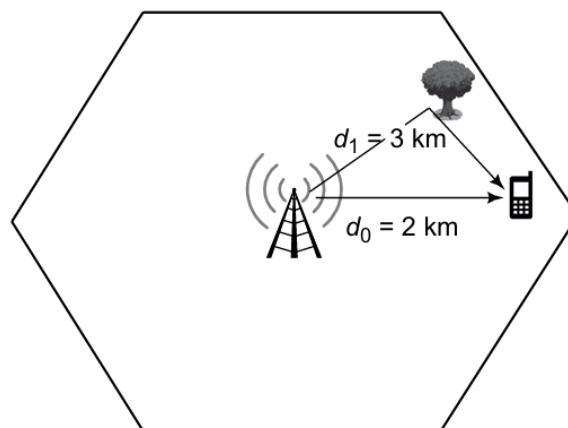


Figure 8: Typical delay spread in outdoor cellular channels

7 Coherence Bandwidth in Wireless Communications

In wireless communication, besides delay spread, another important parameter is the **coherence bandwidth** (B_c). It tells us the frequency range over which the channel's frequency response remains approximately flat.

Definition

The frequency response of the channel is given by:

$$H(f) = \int_0^{\infty} h(\tau) e^{-j2\pi f\tau} d\tau$$

where $h(\tau)$ is the channel impulse response.

B_c is the range of frequencies over which $|H(f)|$ does not change significantly.

Case 1: Zero Delay Spread ($\sigma_\tau = 0$)

If the delay spread is zero, the channel has only a single path:

$$h(\tau) = \delta(\tau)$$

This gives:

$$H(f) = 1$$

Meaning the channel passes all frequencies equally — an **ideal flat-fading channel**.

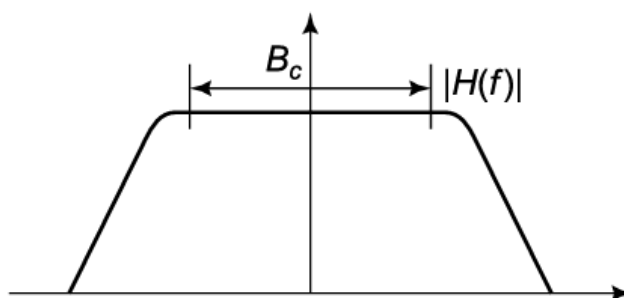


Figure 9: Flat frequency response when delay spread = 0.

Case 2: Non-Zero Delay Spread

As the delay spread increases, the time-domain response spreads, and the frequency response bandwidth decreases. *Longer multipath delay → Narrower frequency range without distortion.*

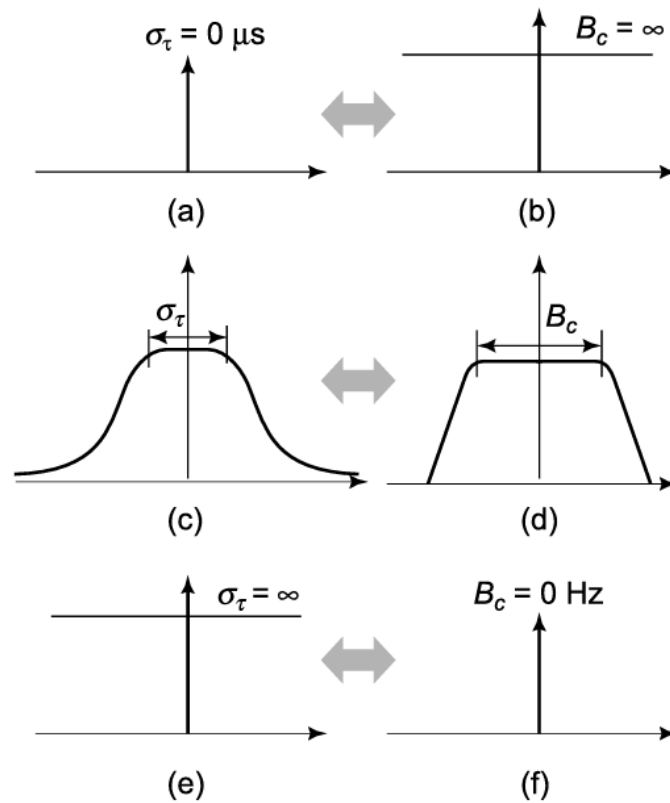


Figure 10: Coherence bandwidth decreases as delay spread increases.

Effect on Transmitted Signals

From system theory:

$$Y(f) = H(f)X(f)$$

where $X(f)$ is the transmitted signal spectrum and $Y(f)$ is the received spectrum.

If $B_s \leq B_c$: The signal lies within the flat part of $H(f) \rightarrow$ no distortion (flat fading).

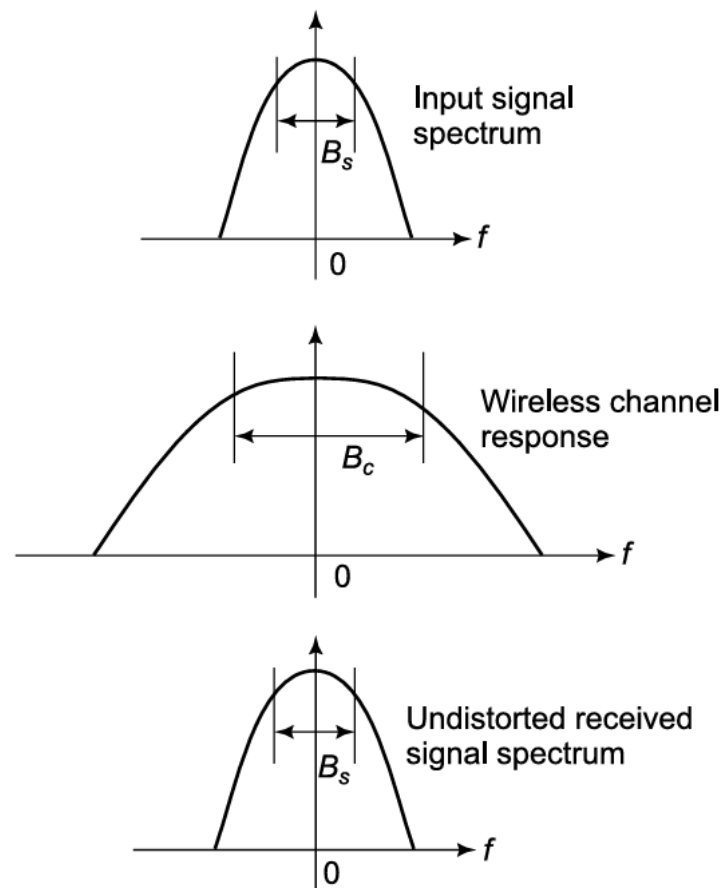
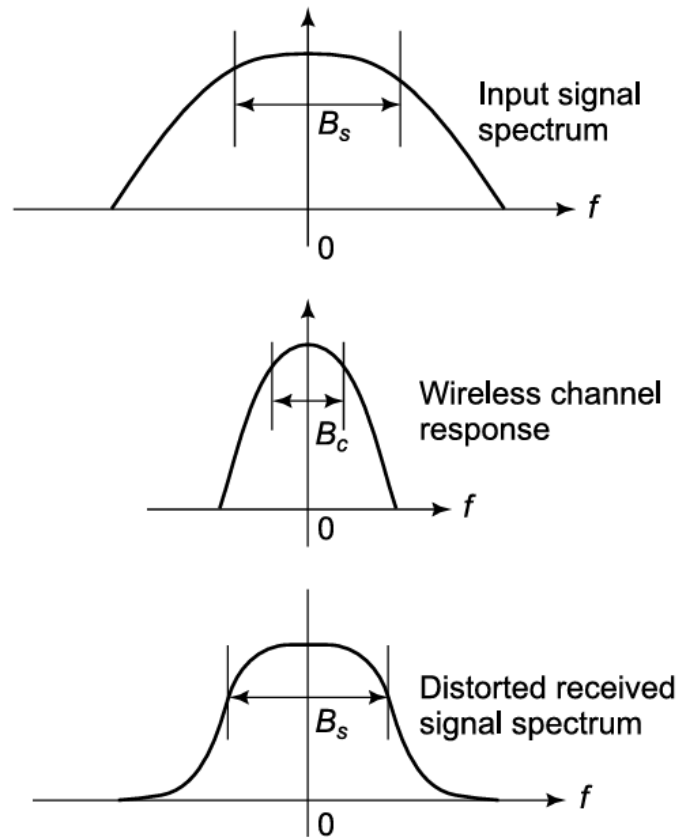


Figure 11: $B_s < B_c$: Flat fading (no distortion).

If $B_s > B_c$: Different parts of $X(f)$ experience different attenuation $\rightarrow$ distortion (frequency-selective fading).

Figure 12: $B_s > B_c$: Frequency-selective fading.

Mathematical Link Between Delay Spread and B_c

For a multipath channel:

$$h(\tau) = \sum_{l=0}^{L-1} a_l \delta(\tau - \tau_l)$$

The frequency where $H(f)$ changes significantly:

$$f_c \approx \frac{1}{4\tau_{L-1}}$$

Thus:

$$B_c \approx \frac{1}{2\tau_{L-1}}$$

If $\tau_{L-1} \approx \sigma_{\tau_{\max}}$ (max delay spread):

$$B_c \approx \frac{1}{\sigma_{\tau_{\max}}}$$

Example: Outdoor Channel

Given $\sigma_\tau = 2 \mu\text{s}$:

$$B_c = \frac{1}{2 \times 2 \times 10^{-6}} = 250 \text{ kHz}$$

So, a channel with $2 \mu\text{s}$ delay spread has a coherence bandwidth of approximately 250 kHz.

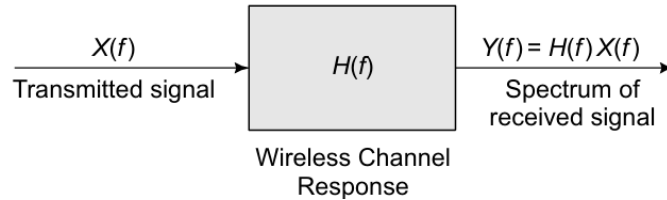


Figure 13: Coherence bandwidth concept in the input-output model.

Inference

- Larger delay spread $\rightarrow$ smaller coherence bandwidth.
- If $B_s \leq B_c \rightarrow$ flat fading (good), else frequency-selective fading (distortion).
- Approximate formula:

$$B_c \approx \frac{1}{\sigma_{\tau_{\max}}}$$

- Example: $2 \mu\text{s}$ delay spread $\rightarrow$ 250 kHz B_c .

8 Relation Between ISI and Coherence Bandwidth

In this section, we explore the relationship between **Inter-Symbol Interference (ISI)** distortion at the receiver and the **coherence bandwidth** B_c of a wireless channel.

Scenario

Consider a Pulse Amplitude Modulated (PAM) signal $x(t)$ with symbol time T_s transmitted by the base station. Assume the channel has:

- A direct line-of-sight path with delay $\tau_0 = 0$.
- A scattered path with delay $\tau_1 = T_d$.

This setup is shown in Figure 14. The received signal is the sum of the direct and scattered components:

$$y(t) = x(t) + x(t - \tau_1)$$

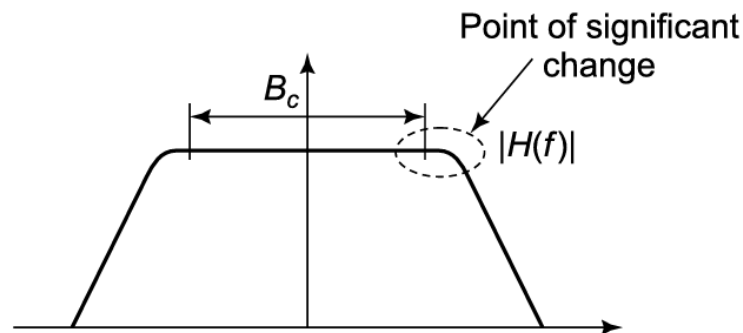


Figure 14: Two-path channel with direct and scattered components.

ISI in the Time Domain

If the delay spread $\sigma_\tau = \tau_1 - \tau_0$ is comparable to the symbol time T_s , then symbols overlap in time. For example, in Figure 15, the symbol s_0 from $x(t)$ overlaps with the previous symbol s_{-1} from $x(t - \tau_1)$.

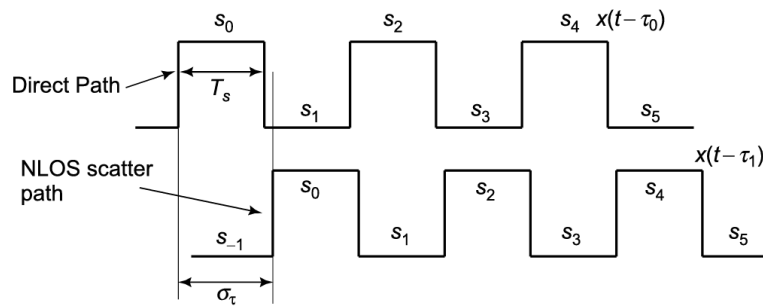


Figure 15: Relation between ISI and delay spread.

As the delay spread increases, more multipath components arrive late, and several symbols interfere, resulting in **severe ISI** (Figure 16).

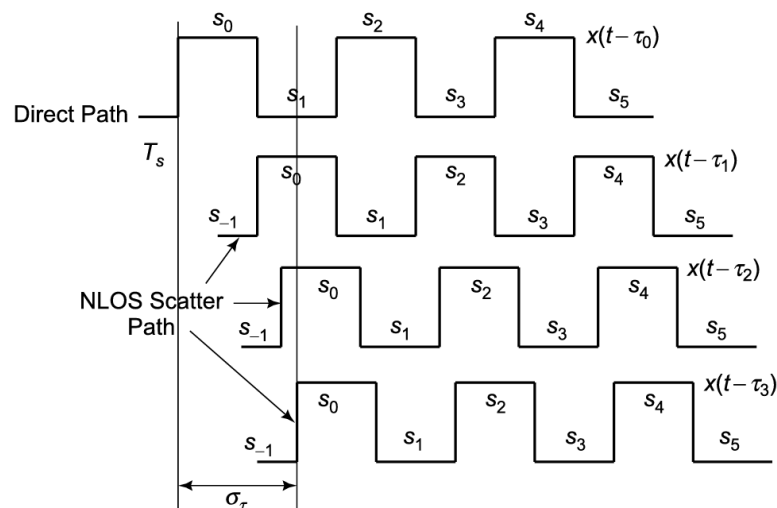
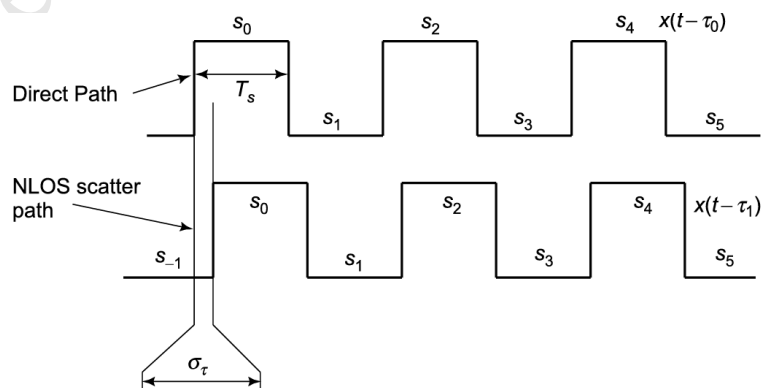


Figure 16: Severe ISI due to multiple scattered components.

When $\sigma_\tau \ll T_s$, ISI is negligible (Figure 17).

Figure 17: Negligible ISI when $\sigma_\tau \ll T_s$.

ISI Criterion in Terms of T_s and T_d

The delay spread T_d can be compared with the symbol time T_s to check for ISI:

$$T_d \geq \frac{1}{2}T_s \Rightarrow \text{ISI occurs.}$$

If $T_s \gg T_d$, no ISI occurs.

Relating to Bandwidth

The symbol time T_s is related to the signal bandwidth B_s by: $T_s = \frac{1}{B_s}$

Also, the delay spread T_d is related to coherence bandwidth B_c by: $B_c \approx \frac{1}{2T_d}$

Substituting $T_d = \frac{1}{2B_c}$ into the ISI criterion: $\frac{1}{2B_c} \geq \frac{1}{2} \cdot \frac{1}{B_s}$

Simplifying: $B_s \geq B_c$

Key Interpretation

- In the **time domain**: If T_d is large compared to T_s , late-arriving paths overlap with later symbols $\rightarrow$ ISI.
- In the **frequency domain**: $B_s \gg B_c$ means the signal's spectrum extends beyond the channel's flat region $\rightarrow$ frequency-selective fading.

Thus, **frequency-selective distortion** and **inter-symbol interference** are essentially two views of the same phenomenon.

Mitigation: Equalization

To combat ISI, the receiver can apply an **equalizer**:

$$H_{eq}(f) = \frac{1}{H(f)}$$

This flattens the channel frequency response, reducing distortion and restoring signal integrity. In later chapters, we will study equalization techniques used in broadband systems such as 2G, 3G, and 4G.

9 Doppler Fading in Wireless Systems

Another unique aspect of communication over wireless channels is **Doppler fading**. This effect arises from the *Doppler shift*, a change in the perceived frequency of an electromagnetic wave due to relative motion between the transmitter and receiver.

- If the transmitter moves **towards** the receiver — the perceived frequency is higher.
- If the transmitter moves **away** — the perceived frequency is lower.
- This is inherent in mobile wireless systems due to the mobility of transceivers.
- In contrast, wired systems do not experience Doppler fading since there is no mobility.

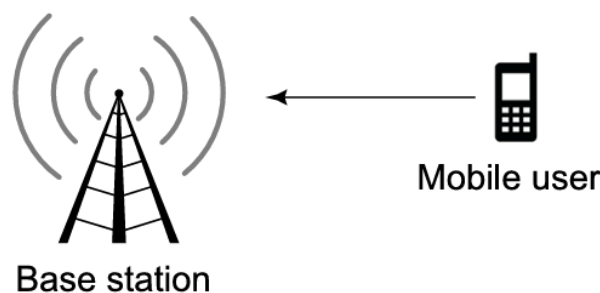


Figure 18: Doppler fading due to user mobility.

9.1 Doppler Shift Computation

Consider a mobile station moving with velocity v at an angle θ to the line between the mobile and base station. Let the carrier frequency be f_c . The Doppler shift is given by:

$$f_d = \frac{v}{c} \cos \theta \cdot f_c$$

where:

- $c = 3 \times 10^8$ m/s (speed of light)
- v = velocity of the mobile
- θ = angle between motion direction and line to receiver

Observations:

- f_d increases with velocity v .
- Maximum f_d occurs when $\theta = 0$ or π (motion along the line of sight).
- $f_d = 0$ when $\theta = \frac{\pi}{2}$ (motion perpendicular to the line of sight).
- Positive f_d means perceived frequency is higher ($0 \leq \theta \leq \frac{\pi}{2}$).

- Negative f_d means perceived frequency is lower ($\frac{\pi}{2} \leq \theta \leq \pi$).

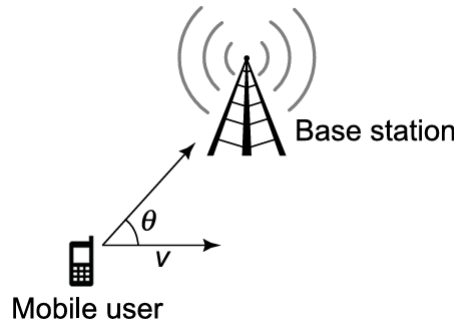


Figure 19: Doppler shift scenario with moving mobile station.

Example

Problem: A vehicle is moving at 60 miles/hour at an angle $\theta = 30^\circ$ with the line joining the base station. Compute the Doppler shift of the received signal at a carrier frequency of $f_c = 1850$ MHz. **Solution:**

1. **Convert velocity to m/s:** We know 1 mile = 1.61 km.

$$v = 60 \text{ mph} = 60 \times 1.61 \text{ km/h}$$

Convert km/h to m/s using $1 \text{ km/h} = \frac{1000}{3600} \text{ m/s} = \frac{5}{18} \text{ m/s}$:

$$v = 60 \times 1.61 \times \frac{5}{18} = 26.8 \text{ m/s}$$

2. **Apply Doppler shift formula:**

$$f_d = \frac{v}{c} \cos \theta \cdot f_c$$

where $c = 3 \times 10^8 \text{ m/s}$. Substituting values:

$$f_d = \frac{26.8}{3 \times 10^8} \times \cos(30^\circ) \times 1850 \times 10^6$$

$$f_d = 143 \text{ Hz}$$

3. **Interpretation:** Since the vehicle is moving **towards** the base station, the Doppler shift is positive. The perceived frequency is:

$$f_r = f_c + f_d = 1850 \text{ MHz} + 143 \text{ Hz}$$

$$f_d = 143 \text{ Hz}, \quad f_r = 1850.000143 \text{ MHz}$$

10 Doppler Impact on a Wireless Channel

In this section, we systematically investigate the impact of Doppler fading on the multipath wireless-channel model.

Consider the impulse response of the i -th component of the multipath channel given as:

$$a_i \delta(t - \tau_i)$$

Let the vehicle be moving with velocity v at an angle θ with respect to the line joining the mobile and base station, as shown schematically in Figure 20.

Because the distance between the base station and the mobile is constantly changing due to the motion of the user, the delay of the i -th signal component is also changing.

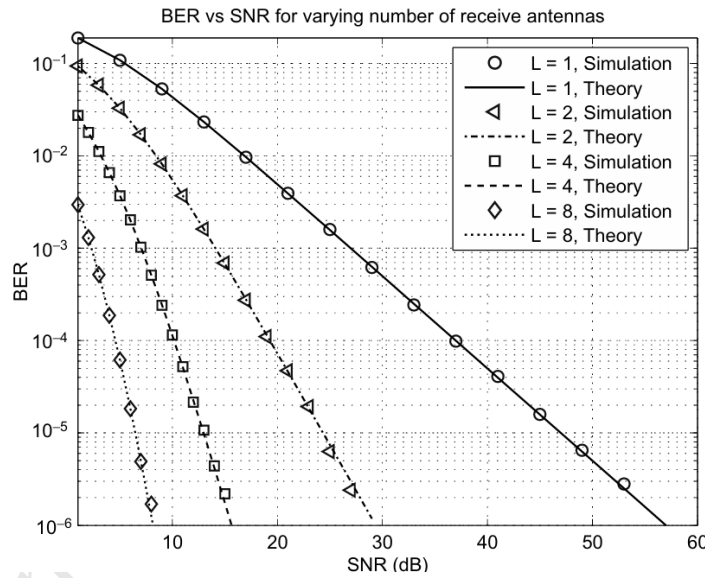


Figure 20: Schematic representation of Doppler impact due to user mobility

Initial Delay

Let the initial distance for the i -th signal component be d_i . The initial propagation delay is:

$$\tau_i = \frac{d_i}{c}$$

where c is the speed of light.

Delay After Time t

After a small interval of time t , the distance decreases by $vt \cos \theta$, since $v \cos \theta$ is the component of the velocity in the direction of the base station.

The delay of the i -th component after time t is:

$$\tau_i(t) = \frac{d_i - vt \cos \theta}{c} = \frac{d_i}{c} - \frac{vt}{c} \cos \theta = \tau_i - \frac{vt}{c} \cos \theta$$

Flat-Fading Wireless Channel Coefficient

Recall from the previous chapter that the flat-fading wireless channel coefficient is defined as:

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

Time-Varying Channel Coefficient

When accounting for user velocity v , we replace τ_i by $\tau_i(t)$:

$$\begin{aligned} h(t) &= \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \left(\tau_i - \frac{v \cos \theta}{c} t \right)} \\ &= \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_c \frac{v \cos \theta}{c} t} \\ &= \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d t} \end{aligned} \quad (4.12)$$

where:

$$f_d = \frac{f_c v \cos \theta}{c}$$

is the **Doppler frequency**.

Interpretation

The term $e^{j2\pi f_d t}$ represents the **time-varying phase** of the wireless channel. The rate of phase variation is given by the Doppler frequency f_d .

- Mobility in a wireless communication system $\Rightarrow$ Doppler shift $\Rightarrow$ time-varying channel coefficient.

- This time-varying nature is termed **time selectivity**.
- A channel can also be **frequency-selective**, meaning different frequencies experience different attenuations.
- A channel that is both time- and frequency-selective is called a **doubly selective channel**.

Department of ECE, ATMECE, Mysuru

11 Coherence Time of the Wireless Channel

Similar to the notion of a **coherence bandwidth** for a frequency-selective channel, we define the concept of a **coherence time** T_c for a **time-varying** channel.

Consider the i -th multipath component of the time-varying channel coefficient in Eq. (4.12):

$$a_i(t) = a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d t}$$

Phase Variation

The value of this i -th component at:

$$\begin{aligned} t = 0 &\Rightarrow a_i(0) = a_i e^{-j2\pi f_c \tau_i} \\ t = \frac{1}{4f_d} &\Rightarrow a_i\left(\frac{1}{4f_d}\right) = a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d \cdot \frac{1}{4f_d}} = j a_i e^{-j2\pi f_c \tau_i} \end{aligned}$$

Thus, empirically, the channel changes significantly from $t = 0$ to $t = \frac{1}{4f_d}$, since the phase changes by $\frac{\pi}{2}$.

Definition

This time duration in which the channel changes significantly due to user mobility is termed the **coherence time**, T_c .

Although f_d depends on the angle of motion θ , a *conservative* estimate (minimum T_c) is obtained by setting $\theta = 0$ (maximum Doppler shift):

$$T_c = \frac{1}{4f_d^{\max}}$$

where:

$$f_d^{\max} = \frac{v}{c} f_c$$

Alternate Expression

The coherence time can also be expressed in terms of the **Doppler spread** B_d :

$$T_c = \frac{1}{2B_d} \quad (4.13)$$